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MATHEMATICS.—*A useful type of formula for the interpolation and representation of experimental results.* L. H. ADAMS. Geophysical Laboratory.

Mellor, in his *Higher Mathematics*,[1] remarks, "The reader will perhaps have been impressed with the frequency with which experimental results are referred to a series formula of the type:

$$y = A + Bx + Cx^2 + Dx^3 + ---,$$

in physical or chemical text books. For instance, I have counted over thirty examples in the first volume of Mendeléeff's *The Principles of Chemistry* and more than this number in Preston's *Theory of Heat*." It is well known that for the representation of experimental results limited power series often leave much to be desired, and that such formulae adequately represent the data only in comparatively few instances or over short ranges; and yet as indicated by the above quotation this fact is seldom appreciated. It is the purpose of this note to point out the advantage of functions other than power series and to advocate their more general employment in cases where power series obviously do not conform to the known general form of curve given by the measurements.

By far the larger number of physical changes follow laws as yet unknown, and so, until the precise theoretical function is ascertained, the experimental results in question can be represented only by some sort of empirical formula. In the choice of

[1] J. W. Mellor, Higher mathematics for students of physics and chemistry, p. 273.

the most suitable equation to be fitted to a given set of experimental results there is usually considerable latitude; but among the host of mathematical functions there will often be certain ones which from their peculiar characteristics or general shape are especially adapted to the data in hand. Nevertheless most chemists and physicists, who desire to fit their experimental results to some equation, turn blindly to the familiar power series. When the graph representing the results is obviously non-linear, the quadratic or parabola, $y = A + Bx + Cx^2$, is first tried; if a satisfactory fit is not obtained another term is added, and so on until there is sufficiently close agreement between the calculated and observed values thruout the range of observation.

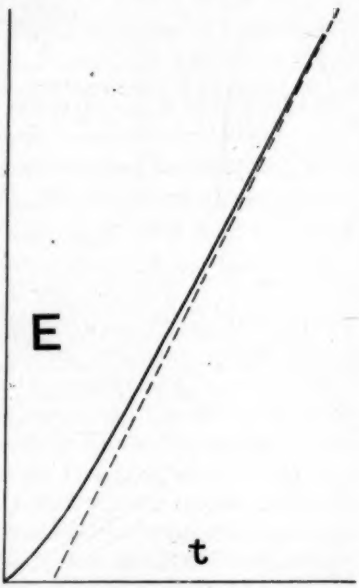

Fig. 1

Now it happens that in some cases a parabolic equation represents the relation between two physical quantities with great exactness. A notable example is the resistance of pure platinum, which is a quadratic function of the temperature between 0° and 1000° to within 0.05 per cent or better. Such accidents, however, are rare. More often than not a cubic or a fourth power equation is necessary especially for experimental observations of high precision. For instance, according to Bridgman[2] a power series of at least five terms—and probably more—would be required to represent the resistance of mercury as a function of pressure, with an accuracy of $\frac{1}{10}$ per cent, the degree of accuracy of the experimental work thruout the range of pressures investi-

[2] Proc. Am. Acad. **44**: 237, (1909).

gated (0–6000 atm.) As a first step in the search for a suitable equation it is advantageous to plot the data and to consider the general form of the graph, especially its limiting characteristics. As an illustration let us consider the e.m.f. of a copper-constantan thermoelement as a function of temperature. If we plot e.m.f. against temperature we obtain a curve of the type shown in figure 1.

The most important characteristic of this curve is its property of becoming more nearly linear the farther it departs from the origin. That is, the curve is asymptotic to a straight line (such as the dotted line shown in the figure), which does not pass

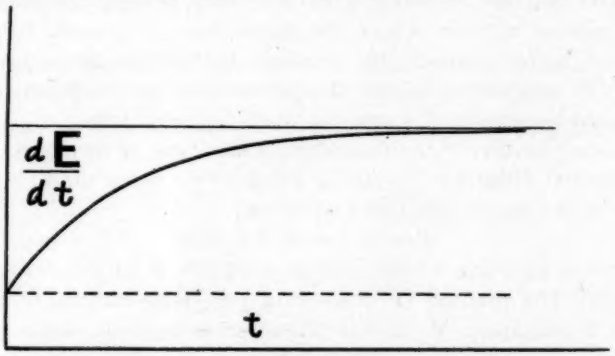

Fig. 2

thru the origin. It is apparent that neither a parabola nor a cubic equation (nor any power series with a reasonable number of terms) can conform over a wide range to this essential condition, namely, that of steadily increasing linearity with increasing values of **E** or t. A power series can be made to approximate only to a greater or smaller portion of the whole curve, the degree of approximation being better the greater the number of terms included in the series and the smaller the portion of the curve dealt with.

On the other hand there are a number of possible functions which have the desired general form. To derive one let us consider the course of the slope $d\mathbf{E}/dt$ of the curve given in figure

1. The derivative is finite at the origin and increases with t, rapidly at first and then more slowly, tending toward a constant value as a limit (see fig. 2). Such a curve referred to the dotted line as the axis is given by $\frac{d\mathbf{E}}{dt} = 1 - e^{-t}$, in general by $\frac{d\mathbf{E}}{dt} = b\,(1 - e^{Ct})$, C being a negative quantity;[3] or when referred to the t – axis by equation $\frac{d\mathbf{E}}{dt} = a + b\,(1 - e^{Ct})$. Integrating and applying the initial condition that $\mathbf{E} = 0$, $t = 0$, we have, $\mathbf{E} = At + B\,(1 - e^{Ct})$. Where $A = a + b, B = \frac{b}{C}$. This equation is of the desired form, and, as we have found by trial, is highly satisfactory in a number of cases where the graph has the general form of figure I, as for instance, the relations between temperature and e.m.f. of copper-constantan thermoelements and between pressure and resistance of manganin or of "therlo" wire.

The calculation of the constants of this type of equation offers no especial difficulty: in fact it involves no more time or labor than in the case of the cubic equation:

$$\mathbf{E} = A't + B't^2 + C't^3$$

a form of equation which is often used and is in general of less utility. The method of evaluating the three constants, A, B and C is as follows: Write down three simultaneous equations containing pairs of corresponding values of $\mathbf{E}$ and t. Eliminate A by addition or subtraction and B by division; we then obtain an equation in C of the form:

$$\frac{(1 - e^{hC}) - P\,(1 - e^{kC})}{(1 - e^{kC}) - Q\,(1 - e^{lC})} = \frac{\mathbf{E}_1 - P\,\mathbf{E}_2}{\mathbf{E}_2 - Q\,\mathbf{E}_3} = K$$

where $\mathrm{P} = \frac{t_1}{t_2}$ and $Q = \frac{t_2}{t_3}$. Assume now several values of C and with the aid of tables of the exponential compute the corresponding values of K. Then plot K against C and determine the proper value of C by graphical interpolation. The value of C being now known, B and finally A are readily calculated.

[3] Another suitable function is $\frac{dE}{dt} = a + b \tanh ct$ which would lead to the equation $E = aT + b \log \cosh ct$.

The result of the application of this equation to the copper-constantan element is shown in table I. The constantan wire consisted of selected lengths cut from a spool of No. 30 B. and S. "Ideal" wire made by the Electrical Alloy Co. The points at 100°, 217°.95 and 305°.9 were obtained by immersing the element (enclosed in a glass tube) in the vapors of boiling water, naphthalene and benzophenone;[4] those at the lower temperatures by comparison with an element[5] previously calibrated against a resistance thermometer. As table I shows, the curve when passed thru the three upper points fits the three lower points very satisfactorily; indeed the divergence is no greater than the experimental error.

TABLE I

COMPARISON OF OBSERVED E. M. F. (E) OF COPPER-CONSTANTAN ELEMENT WITH THAT CALCULATED FROM THE EQUATION $E = 74.672\,t - 13892\,(1 - e^{-0.00261\,t})$

TEMP.	E (OBS.) MICROVOLTS	E (CALC.) MICROVOLTS	DIFFERENCE[1] MICROVOLTS
24°.30	961.1	960.9	0.2
49°.59	2010.9	2010.8	0.1
78°.75	3291.7	3291.4	0.3
100°.00	4276.	(4276.)	...
217°.95	10248.	(10248.)	...
305°.9	15203.	(15203.)	...

[1] 1 microvolt corresponds to about 0°.02.

The above equation as compared with a series formula containing a like number of constants (i.e., a cubic equation) possesses the following advantages, (1) In accord with what might be expected from the general shape of the curve a better agreement has been obtained between "observed" and "calculated" values. For instance if a cubic should be substituted for the exponential equation described, the differences in the fourth column of table I would each be greater than 5 microvolts (0°.1). (2) Since values of $(1 - e^{-x})$ may be obtained by inspec-

[4] Proper correction being made for barometric pressure. Cf. Adams & Johnston, Am. J. Sci., **33**: 538 et seq. 1912.

[5] This element belongs to Dr. W. P. White of this Laboratory. For description of the comparison of this element with the resistance thermometer, see Phys. Rev. **31**, 159. 1910.

tion and in one operation from tables of the Descending Exponential,[6] the operation of calculating a series of values from the exponential equation involves less labor than from a cubic equation. (3) Altho extrapolation is, in general, an operation beset with many pitfalls, nevertheless, if an extrapolation is unavoidable, it is more reasonable to employ a function of proper general form than to take some random function which can not be expected to fit the experimental results except over a small part of the range of observation.

MINERALOGY.—*Hodgkinsonite, a new mineral from Franklin Furnace, N. J.* C. PALACHE, Harvard University, and W. T. SCHALLER, Geological Survey.

The mineral here described was sent to the Harvard Mineralogical Museum for identification in April of this year by Mr. J. J. McGovern of Franklin Furnace, New Jersey. On being informed that the mineral was of a new species, material for further study and for analysis was freely supplied by Mr. H. H. Hodgkinson, M. E., Assistant Underground Superintendent of the mine, who first found the mineral in the mine workings and whose name it bears. Mr. Hodgkinson states that the new mineral was found in the northern part of the ore body, in that part of the Parker Mine formerly known as the Hamburg Mine and quite near the hanging wall of the west leg of the ore body, between the 850- and 900-foot levels. The locality was marked by a number of slips and faults, along some of which the mineral occurs. It has been found in a number of specimens during the year but nowhere in abundance.

Hodgkinsonite is a hydrous silicate of zinc and manganese crystallizing in the monoclinic system. It occurs in seams in massive granular ore of the typical willemite-franklinite mixture; the seams are generally very thin with but a film of the mineral which is always associated with white barite and not uncommonly with plates of native copper. Locally the film thickens to a narrow vein and then the new mineral may show individuals up to 2 cm. across, sharply angular in form and apparently with

[6] Such, for example, as those in Becker and Van Ostrand's hyperbolic functions (Smithsonian Mathematical Tables, Washington. 1909).

crystal faces but in reality determined in their outline almost wholly by the older platy barite which encloses them. The clear pink color and brilliant cleavage of hodgkinsonite, together with the snow white barite make such specimens both striking and attractive in appearance. One mass of ore with a surface 20 cm. square is at least half covered with hodgkinsonite. In one case only has such a vein been found in which the angular cells formed by the intersecting barite plates were not wholly filled by hodgkinsonite so that the latter was free to develop crystal planes. From this specimen three crystals of good quality were detached and these served to establish the axial ratio of the species.

Other crystals were found occupying cavities in thicker veins free from barite. These crystals, the largest 1 cm. long, were much affected by solution, the faces being generally dull or facetted. They were accompanied by black rhombohedral crystals of pyrochroite and scalenohedral crystals of calcite, both later in age and encrusting hodgkinsonite. The latter is implanted directly on willemite or franklinite and in one specimen on manganese garnet. The association and mode of occurrence both indicate a pneumatolytic origin for the new mineral.

Hodgkinsonite is monoclinic with normal symmetry. The highly perfect cleavage, normal to the symmetry plane, has been taken as the basal pinacoid. The elements were calculated from the angles of the forms taken as (110), (011), and (221) together with the inclination of the cleavage (001) to the prism zone. These angles follow.

(001) $\left\{\begin{array}{l}\varphi = 90°\ 00' \\ \rho = 5°\ 27\frac{1}{2}'\end{array}\right\}$ Average of 2 readings on 2 crystals.

(110) $\left\{\begin{array}{l}\varphi = 33°\ 10' \\ \rho = 90°\ 00'\end{array}\right\}$ Average of 7 readings on 3 crystals.

(011) $\left\{\begin{array}{l}\varphi = 4°\ 42' \\ \rho = 48°\ 16'\end{array}\right\}$ Average of 2 readings on 1 crystal.

(221) $\left\{\begin{array}{l}\varphi = 34°\ 48' \\ \rho = 69°\ 48'\end{array}\right\}$ Average of 4 readings on 2 crystals.

The axial elements thus calculated are:

$p_0 = 0.7254 \quad q_0 = 1.1114 \quad \mu = 84°\ 33\frac{1}{2}'.$
$a : b : c = 1.539 : 1 : 1.1165 \quad \beta = 84°\ 33\frac{1}{2}'.$

The forms observed are $c(001)$, $m(110)$, $s(011)$, $r(221)$, $t(\bar{4}01)$, $x(\bar{3}05)$, $q(552)$, $u(\bar{3}22)$. The last four of these forms were found only on the etched crystals, which had been measured with considerable care before the better crystals were found. These four high-index forms are no doubt vicinal and due to etching of the crystals; they are nevertheless retained in the table of measurements and a figure showing them is given since they are characteristic of most of the specimens.

In the following table may be found all the observations made on these crystals, together with the calculated angles for the various forms.

TABLE 1

ANGLE TABLE FOR HODGKINSONITE

	CALCULATED		MEASURED		LIMITS		NO. OF FACES
	φ	ρ	φ	ρ	φ	ρ	
$c(001)$....	90°00′	5°27½′	90°00′	5°27½′		5°27′– 5°28′	2
$m(110)$...	33 08	90 00	33 10	90 00	33°08′–33°11′		7
$s(011)$....	4 53½	48 15½	4 52	48 14	3 34 – 6 23	47 28 –48 47	12
$r(221)$....	34 49	69 49	34 48	69 48	34 45 –34 51	69 35 –69 59	4
$t(\bar{4}01)$....	–90 00	70 28	–90 00	70 09		70 03 –70 15	2
$x(\bar{3}05)$....	–90 00	18 52	–90 00	18 13		17 36 –18 35	3
$q(552)$....	34 29	73 33	34 30	72 34	34 01 –35 10	71 45 –73 23	4
$u(\bar{3}22)$...	–41 47	56 16	–43 15	55 25	42 59 –43 31		2

The crystals are acute pyramidal in habit as shown in figure 1, and are dominated by the equal development of unit prism m and pyramid r. On the etched crystals this pyramid is replaced by a group of facets whose average position corresponds to the symbol (552). The clinodome and prism faces are always smooth but often dull and poorly reflecting. The faces of other forms are minute and of the poorest quality. Figure 2 shows one of the etched crystals, doubly terminated and with the small faces of uncertain forms characteristic only of specimens of this type.

The perfect cleavage of hodgkinsonite is parallel to the basal pinacoid. The density is 3.91, determined by a pycnometer on

a gram of small fragments which were later used for analysis. The hardness is a very little less than 5.

The optical characters have been only imperfectly determined. The optic axial plane is parallel to (010). The mean refractive index is 1.73, determined by the immersion method. The color of the mineral varies from a bright pink to a pale reddish brown, the luster is vitreous, the streak white.

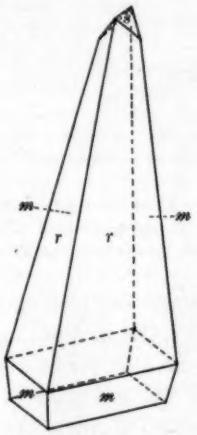

Fig. 1

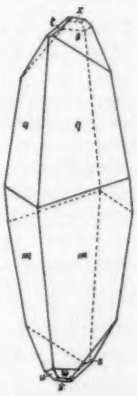

Fig. 2

Fig. 1. Hodgkinsonite. Common habit of implanted crystals, showing basal cleavage when removed from the matrix. Forms $m(110)$, $s(011)$, $r(221)$. On etched crystals r is replaced by $q(552)$ without change of habit.

Fig. 2. Hodgkinsonite. Doubly terminated etched crystal. Forms: $c(001)$, $m(110)$, $s(011)$, $x(\bar{3}05)$, $q(552)$, $u(\bar{3}22)$.

The mineral decrepitates when held in the blowpipe flame, wherein it fuses readily and quietly to a brown enamel. Heated in a closed tube, hodgkinsonite decrepitates strongly, splitting up into numerous thin cleavage scales which on further heating yield water and become brown in color. The mineral is readily soluble in acid, yielding gelatinous silica.

The chemical analysis of hodgkinsonite was made on carefully selected cleavage fragments of a clear pink color and gave the

following results. Quarter gram samples were used as the total amount of available material was only somewhat over a gram.

ANALYSIS AND RATIOS OF HODGKINSONITE (BY W. T. SCHALLER)

	1	2	3	AVERAGE	RATIOS	
SiO_2	19.92	19.89	19.77	19.86	0.331	1.02
MnO	20.39	20.97		20.68	0.291	
ZnO	52.93	*		52.93	0.653	2.98
CaO	0.99	0.88		0.93	0.017	
MgO	0.04			0.04	0.001	
H_2O			5.77†	5.77	0.321	0.99
				100.21		

* A duplicate determination of ZnO, of which a small amount was lost, gave 51.38 per cent.

† Determined directly by fusing the mineral with sodium carbonate and collecting the water in a calcium chloride tube. A determination of the loss on ignition, corrected for oxidation of the manganese, gave the value 4.68 per cent.

A doubtful trace of lead was encountered but iron and chlorine were absent. No water was given off by the mineral at 110°.

The ratios yield sharply the formula 3 $RO.SiO_2.H_2O$ where R is chiefly zinc and manganese. If the manganese, calcium and magnesium be arbitrarily taken together then the ratio of MnO + CaO + MgO to ZnO is 309 to 653 or 1 : 2.11 or nearly 1 : 2. The formula may then be written $MnO.2ZnO.SiO_2.H_2O$ which may, as the water is all constitutional, be interpreted as $Mn.(ZnOH)_2.SiO_4$.

PROCEEDINGS OF THE ACADEMY AND AFFILIATED SOCIETIES

THE ANTHROPOLOGICAL SOCIETY OF WASHINGTON

A special meeting of the Society was held on October 28, 1913, at the National Museum.

DR. ALES HRDLICKA addressed the Society, his subject being *The results of the speaker's recent trip to Peru; with remarks on the anthropological problems of Peru*, illustrated with lantern slides. In 1910 the speaker made a brief exploratory trip in Peru, which resulted in the acquisition of some valuable data and of important skeletal collections. The opportunity to extend the investigations came during the early part of the current year, in connection with the preparation of the anthropological exhibits for the Panama-California Exposition at San Diego; and as a consequence three busy months were spent on the Peruvian coast and in certain parts of the mountain region of Peru, in exploration of the ruined cities and ancient cemeteries. The principal objects of the trip were, first, the mapping out as far as possible of the anthropological distribution of the prehistoric Peruvian, more particularly the coast people; second, the determination of the physical type of the important Nasca group of people, which represent one of the highest American cultures; third, further inquiry as to man's antiquity on the west coast of South America; and fourth, the extension of the speaker's researches on pre-Columbian pathology. The conclusions to which the speaker was formerly led were in the main corroborated. In regard to the mountain regions much remains to be determined in the future. As to the pathology of the native Peruvian before contact with whites, the main work can perhaps be now regarded as done, or nearly so, altho individual variation in different morbid processes seems inexhaustible, and much in this line remains to be secured by future exploration. The ground covered was extensive and the skeletal material examined was enormous, the selections alone filling over thirty boxes. No excavation was practiced, attention being restricted, on the coast, to the bones covering the surface of ancient cemeteries, exploited by the peons, and to burial caves and houses in the mountains.

Since the speaker's trip to Peru three years ago, a change for the worse was observed in the state of preservation of the ancient remains. Also, where formerly there were seemingly inexhaustible quantities of skeletal material there is now a dearth of it. No such collection as that made in 1910, when the speaker gathered 3400 important crania, will ever again be possible from these regions. The major part of the

old population of the coast region belongs to the brachycephalic type intimately related to the Maya Zapotec type in the north. Wherever they lived, these people of the Peruvian coast were wont to practice, more or less, the antero-posterior head deformation. Everywhere along the coast there are evidences of more or less admixture with a more oblong headed element closely related to the Aztec and Algonquin types of North America. As among the North American Pueblos, nowhere was the aboriginal Peruvian population at any time as great as the relatively numerous cemeteries or ruins might lead one at first to suppose, for these burial grounds and ruins date from different, altho not far distant, periods.

The work done, while to some extent establishing a foundation, is merely a fair beginning. Similar investigations and collections wait urgently on the anthropologist in the important districts of Piura, Eton, and Moquegua, on the coast; in the western sierras from the neighborhood and latitude of Cajamarca to those of Arequipa; and in the eastern highlands from Tiahuanaco to Moyobamba. The most important problems that await solution are (1) the derivation of the Peruvians; (2) the time of their advent into the country; (3) the extension and exact physical characteristics of the Aymara and Quechua; and (4) the genetic relations of the Peruvian to the Argentinan and Chilean aborigines. Besides this there remains to be established in many places the correlation of culture with the physical type of the people. The speaker repeated what he said in a former report, that, due to the lack of scientific supervision of a great majority of the excavations practiced in Peru to the present time, the archeological collections from that country are made up of little more than curiosities which it is impossible to refer either to any definite people or period.

DANIEL FOLKMAR, *Secretary.*